6-9 The Tangent Ratio

What You'll Learn

1. To find the tangent ratio
2. To solve problems using the tangent ratio

...And Why

You can use the tangent ratio to find measures indirectly.

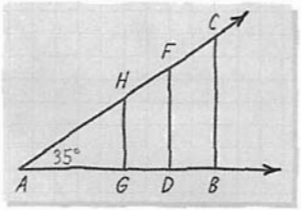
Here's How

Look for questions that

- build understanding
- ✓ check understanding

Work Together — *Exploring the Tangent Ratio*

1. a. Using graph paper and a protractor, draw $\angle A$, a 35° angle with one side on a horizontal line. Then choose vertical lines to draw segments $\overline{HG}$, $\overline{FD}$, and $\overline{CB}$ as shown.



   b. Use a centimeter ruler to measure the segments in the first two columns below to the nearest millimeter. Use a calculator to evaluate the ratios in the third column.

$BC =$ ■	$AB =$ ■	$\frac{BC}{AB} =$ ■
$DF =$ ■	$AD =$ ■	$\frac{DF}{AD} =$ ■
$GH =$ ■	$AG =$ ■	$\frac{GH}{AG} =$ ■

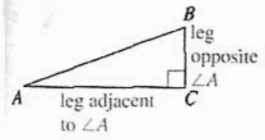
   c. *Patterns* What pattern do you notice in the ratios in the table? Compare the ratios you found with those of other members of your group.

2. Repeat Question 1, but this time, draw the figure so that $m\angle A = 62°$.

THINK AND DISCUSS

1 Finding the Tangent Ratio

In the Work Together, you began the study of trigonometry, which comes from the Greek words meaning "triangle measurement." The ratio that you calculated is called the tangent ratio of $\angle A$. You discovered that the tangent ratio remains the same for any right triangle with the same measure of $\angle A$.

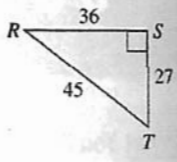


tangent of $\angle A = \dfrac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A}$

You can abbreviate this equation as $\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$.

■ EXAMPLE 1

In $\triangle RST$ at the right, find $\tan T$.



$$\tan T = \frac{\text{length of leg opposite } \angle T}{\text{length of leg adjacent to } \angle T}$$
$$= \frac{36}{27}$$
$$= \frac{4}{3} = 1.\overline{3}$$

3. ✓ *Try It Out* Write $\tan R$ as a fraction and as a decimal.

4. *Reasoning* How is $\tan R$ related to $\tan T$?

You can use a calculator to find the tangent of any acute angle of a right triangle. To find tan 59°, use this key sequence.

59 TAN 1.6642794 So, $\tan 59° \approx 1.6643$.

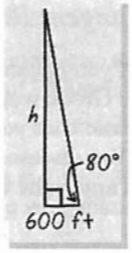
5. *Calculator* Use a calculator to find each tangent. Round to the nearest ten-thousandth.
   a. tan 13° b. tan 47° c. tan 86°

2 Solving Problems Using the Tangent Ratio

You can use the tangent ratio to find measures indirectly.

■ EXAMPLE 2 *Real-World Problem Solving*

Rock Climbing In Yosemite National Park in California, a granite mass named El Capitan rises almost perpendicular to the valley floor. When you are standing 600 feet from the wall, you look up at an angle of 80° and see a rock climber near the top of El Capitan. Use indirect measurement to estimate the height of the climber.



Draw a diagram to show the given measures. Let h represent the height of the climber.

$\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$ ← Use the tangent ratio.

$\tan 80° = \dfrac{h}{600}$ ← Substitute.

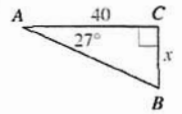
$h = 600(\tan 80°)$ ← Solve for h.

600 × 80 TAN = 3402.7690

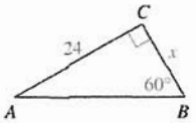
The climber is about 3,400 feet above the ground.

6. ✓ *Try It Out* Find the value of x to the nearest tenth.

a.



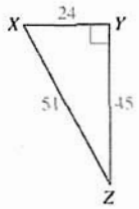
b.



7. *Go a Step Further* Use your answers to Question 6 to find the length of the hypotenuse of each triangle. Round your answers to the nearest tenth.

EXERCISES *On Your Own*

1. Refer to $\triangle XYZ$ at the right.
   a. Which side is opposite $\angle X$? Opposite $\angle Z$?
   b. Which side is adjacent to $\angle X$? Adjacent to $\angle Z$?
   c. Find tan X and tan Z as fractions in simplest form and as decimals.
   d. What is the relationship between tan X and tan Z?

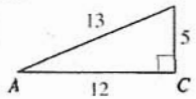


Use a calculator to find each tangent. Round to the nearest ten-thousandth.

2. tan 18°
3. tan 85°
4. tan 22°
5. tan 45°

Find tan A and tan B in each figure below. Express your answers as fractions in simplest form.

6.



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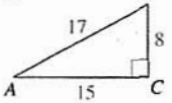

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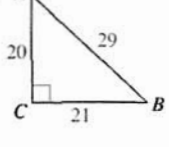

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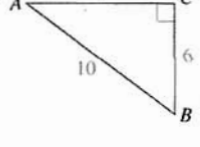
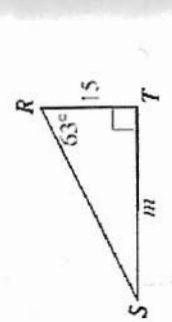
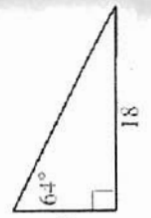
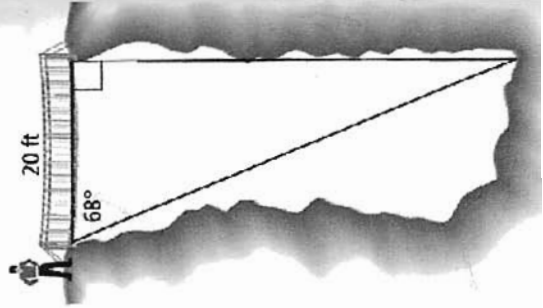


10. $\triangle PQR$ is a right triangle with $PQ = 25$, $PR = 60$, and $QR = 65$. Find tan Q as a fraction in simplest form and as a decimal. (*Hint:* The longest side of a right triangle is the hypotenuse.)

11. $\triangle ABC$ is a right triangle with $AB = 53$, $BC = 45$, and right angle C. Draw a diagram of $\triangle ABC$ and find tan A and tan B as fractions in simplest form. (*Hint:* Use the Pythagorean theorem.)



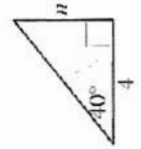




12. Refer to $\triangle RST$ at the right.
   a. Find m by writing and solving an equation involving tan R.
   b. Find m by writing and solving an equation involving tan S.
   c. *Writing* Which method do you prefer? Explain why.

In each figure, find n. Round to the nearest tenth.

13.



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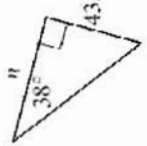

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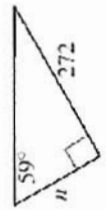

16.

17. *Surveying* A surveyor stands at the end of a bridge across a canyon. The bridge is 20 feet long. The surveyor looks down at an angle of 68° to see the bottom of the canyon on the opposite side. Find the depth of the canyon to the nearest foot.

18. *Communications* A 200-m tower is to be built for relaying cellular phone signals. The tower is to be anchored by cables from the top of the tower that will each form a 65° angle with the ground. Find how far from the base of the tower each cable will be anchored. (*Hint:* Each cable forms the hypotenuse of a right triangle.)

19. *Open-ended* Make up a problem that you can solve using the tangent ratio. Write a solution for your problem.